## ACT Mathematics Test: Strategies and Concept Review

The ACT Mathematics Test is designed to test your ability to reason mathematically, to understand basic mathematical terminology, and to recall basic mathematical formulas and principles. You will have 60 minutes to complete the ACT Mathematics Test. You should be able to solve problems and apply relevant mathematics concepts in the following areas:

**Preparing for Higher Math** (Approximately 60% of the questions) This category has five subcategories.

•   **Number & Quantity** (Approximately 10% of the questions)

Real and complex number systems

Integer and rational exponents

Vectors

Matrices

•   **Algebra** (Approximately 15% of the questions)

Linear expressions

Polynomials

Radicals

Exponential relationships

•   **Functions** (Approximately 10% of the questions)

Function definition, notation, representation, and application

Linear, radical, piecewise, polynomial, and logarithmic functions

•   **Geometry** (Approximately 15% of the questions)

Congruence, similarity relationships, surface area

Volume measurements

Triangles and circles

Trigonometric ratios

Equations of conic sections

•   **Statistics & Probability** (Approximately 10% of the questions)

Center and spread of distributions

Data collection methods

Bivariate data

Probabilities

**Integrating Essential Skills** (Approximately 40% of the questions)

Rates and percentages

Proportional relationships

Area, surface area, and volume

Average and median

### ****GENERAL STRATEGIES AND TECHNIQUES****

Use the following general strategies when tackling the ACT Mathematics Test.

**Steve Says:**

Be sure to practice the strategies and techniques covered in this chapter on the simulated tests found in Part IV of this book.

**Draw Pictures**

It really helps sometimes to visualize the problem. This strategy should not take a lot of time and can prevent careless errors. Your sketches can be quick and even a little messy. Sometimes they give you a picture; sometimes you have to just make your own.

Consider the following example:

In the xy-coordinate plane, point P is at (2, 3), point R is at (8, 3), and point S is at (8, 6). What is the area of triangle PRS?

**A.**   4.5

**B.**   9

**C.**   13.5

**D.**   18

**E.**   27

The area of a triangle is  (base) (height). To solve this problem it might be helpful to draw a diagram, as shown below:

The line segment PR is 6 units long (because the distance between x-coordinates is 6 units), and it is the base of the right triangle. The line segment RS is 3 units long, and it is the height of the right triangle. Using this information, the area of triangle PRS is  (6)(3), or 9.

**Think Before Computing**

Most of the calculations are fairly simple and actually will not require the use of a calculator. In fact, the ACT test writers are just as likely to test your logical reasoning ability or your ability to follow directions as they are to test your ability to punch information into your calculator. If you do use your calculator, be sure that you have a good idea of what your answer should look like ahead of time. If the answer you get from your calculator is not at least in your expected ballpark, try again.

Consider the following example:

If b − c = 2, and a + c = 16 then a + b = ?

**A.**   8

**B.**   14

**C.**   16

**D.**   18

**E.**   32

To solve this problem, first recognize that (b − c) + (a + c) = a + b. This is true because the c values cancel each other out, leaving you with b + a, which is equivalent to a + b. Therefore, a + bmust equal 2 + 16, or 18.

Alternatively, you could solve the first equation for c and substitute the solution into the second equation, as follows:

b − c = 2

c = b − 2

a + c = 16

a + (b − 2) = 16

a + b = 18

**Answer the Question That They Ask You**

If the problem requires three steps to reach a solution and you only completed two of the steps, it is likely that the answer you arrived at will be one of the choices. However, it will not be the correct choice! Don’t quit early—reason your way through the problem so that it makes sense. Keep in mind, though, that these questions have been designed to take an average of 1 minute each to complete. They do not involve intensive calculations.

Consider the following example:

The rectangular garden shown in the figure has a stone border 2 feet in width on all sides. What is the area, in square feet, of that portion of the garden that excludes the border?

**A.**   4

**B.**   16

**C.**   40

**D.**   56

**E.**   72

This problem is asking for the area of the middle portion of the garden. To solve this problem, perform the following calculations, and remember that the border goes around the entire garden. First, subtract the border width from the length of the garden:

12 − 2(2) = 8

Next, subtract the border width from the width of the garden:

6 − 2(2) = 2

The area (length × width) of the portion of the garden that excludes the border is 8 × 2, or 16.

If you only accounted for the border along one length and one width of the garden, you would have gotten answer choice C. Answer choice D is the area of the border around the garden. Answer choice E is the area of the entire garden, including the stone border.

**Check the Choices**

Take a quick peek at the choices as you read the problem for the first time. They can provide valuable clues about how to proceed. For example, you may be able to substitute answer choices for variables in a given equation.

Consider the following example:

If 0 < pr < 1, then which of the following CANNOT be true?

**A.**   p < 0 and r < 0

**B.**   p < −1 and r < 0

**C.**   p < −1 and r < −1

**D.**   p < 1 and r < 1

**E.**   p < 1 and r < 0

At first glance, you might think that you don’t have enough information to solve this problem. However, if you recognize that pr must be a positive fraction since it lies between 0 and 1, you can work your way through the answer choices and eliminate those that could be true:

**Answer choice A:** If both p and r were less than 0, their product would be positive. It’s possible for pr to be a positive fraction because both p and r could be negative fractions, so eliminate answer choice A.

**Answer choice B:** If p were –1 and r were also a negative number their product would be positive. It’s possible for pr to be a positive fraction because r could be a negative fraction, so eliminate answer choice B.

**Answer choice C:** If both p and r were less than –1, then pr would be greater than 1, so this statement cannot be true, and answer choice C is correct.

**Answer choice D:** If both p and r were less than 1, their product could be positive. It’s possible for pr to be a positive fraction because both p and r could be negative fractions, so eliminate answer choice D.

**Answer choice E:** If p were less than 1, p could be a positive fraction. If r were greater than 0, it would be a positive number, and it’s possible for pr to be a positive fraction; eliminate answer choice E.

**Amy Says:**

You don’t get any extra points for answering the harder questions. So, do not waste time on a question when you aren’t making any progress. Go find some questions that are easier for you and come back to the tougher ones only if you have time.

**Test the Answers**

Sometimes the quickest way to answer an ACT math question is to try the answer choices that they give you. The questions on the ACT Mathematics Test have five answer choices each, and the numerical choices are arranged in ascending or descending order. This means that if you are “trying out” answer choices, it makes sense to try the middle value (choice C or choice H) first. If the middle value is too small, you can eliminate the other two smaller choices. And, if it is too large, you can eliminate the other two larger choices.

Consider the following example:

Which of the following is a value of x for which (x − 3)(x + 3) = 0?

**A.**   2

**B.**   3

**C.**   5

**D.**   6

**E.**   7

One approach to answering this question is to try the answer choices. Start with answer choice C:

(5 − 3)(5 + 3) = (2)(8) = 16

Answer choice C results in an answer that is too big. Because answer choices D and E are both larger than answer choice C, they will result in answers that are greater than 16. Therefore, you can eliminate answer choices C, D, and E, simply by trying answer choice C. Now try answer choice B:

(3 − 3)(3 + 3) = (0)(6) = 0; answer choice B is correct.

**Use “Stand-Ins”**

You can sometimes simplify your work on a given problem by using actual numbers as “stand-ins” for variables. This strategy works when you have variables in the question and some of the same variables in the answer choices.

You can simplify the answer choices by substituting actual numbers for the variables. If you use this strategy, remember that numbers on the ACT can be positive or negative and are sometimes whole numbers and sometimes fractions. You should also be careful not to use 1 or 0 as your stand-ins because they can create “identities,” which can lead to more than one seemingly correct answer choice.

Consider the following example:

If a and b are positive consecutive odd integers, where b > a, which of the following is equal to b2 − a2?

**A.**   2a

**B.**   4a

**C.**   2a + 2

**D.**   2a + 4

**E.**   4a + 4

You are given that both a and b are positive consecutive odd integers, and that b is greater than a. Pick two numbers that fit the criteria: a = 3 and b = 5. Now, substitute these numbers into b2 − a2 : 52 = 25 and 32 = 9; therefore, b2 − a2 = 16. Now, plug the value that you selected for a into the answer choices until one of them yields 16, as follows:

2(3) = 6; eliminate answer choice A.

4(3) = 12; eliminate answer choice B.

2(3) + 2 = 8; eliminate answer choice C.

2(3) + 4 = 10; eliminate answer choice D.

4(3) + 4 = 16; answer choice E is correct.

**Simplify the Question**

Some ACT questions appear quite difficult at first glance, but after careful review, turn out to be pretty straightforward questions. They often include formulas and equations, etc., and ask you to solve for a variable.

Consider the following example:

Each student’s project in an art class is given a point score by the teacher and by each of the other students in the class. A student’s project grade, g, is determined by the formula , where t is the score the teacher gives, s is the sum of the scores the students give, and n is the number of students in the class. What is t in terms of g, s, and n?

**A.**   t = g – n – s

**B.**   t =

**C.**   t = gn + g – s

**D.**   t =

**E**.

To solve this problem, let the answer choices guide you. Note that each answer choice is a solution for t, so all you need to do is solve the given formula for t, as follows:

### MATHEMATICS CONCEPT REVIEW

This section serves as a review of the mathematical concepts tested on the ACT. Familiarize yourself with the basic mathematical concepts included here and be able to apply them to a variety of math problems.

**Pre-Algebra**

The Pre-Algebra (seventh- or eighth-grade level) questions test basic algebraic concepts such as:

1.   Operations Using Whole Numbers, Fractions, and Decimals

2.   Square Roots

3.   Exponents

4.   Scientific Notation

5.   Ratios, Proportions, and Percent

6.   Linear Equations with One Variable

7.   Absolute Value

8.   Simple Probability

**Operations Using Whole Numbers, Decimals, and Fractions**

The ACT Mathematics Test will require you to add, subtract, multiply, and divide whole numbers, fractions, and decimals. When performing these operations, be sure to keep track of negative signs and line up decimal points in order to eliminate careless mistakes.

The following are some simple rules to keep in mind regarding whole numbers, fractions, and decimals:

1.   Ordering is the process of arranging numbers from smallest to greatest or from greatest to smallest. The symbol > is used to represent “greater than,” and the symbol < is used to represent “less than.” To represent “greater than or equal to,” use the symbol ≥; to represent “less than or equal to,” use the symbol ≤.

2.   The Commutative Property of Multiplication is expressed as a × b = b × a, or ab = ba.

3.   The Distributive Property of Multiplication is expressed as a(b + c) = ab + ac.

4.   The order of operations for whole numbers can be remembered by using the acronym **PEMDAS**:

**P**        First, do the operations within the **parentheses**, if any.

**E**        Next, do the **exponents**.

**MD**    Next, do the **multiplication** and **division**, in order from left to right.

**AS**      Finally, do the **addition** and **subtraction**, in order from left to right.

5.   When a number is expressed as the product of two or more numbers, it is in factored form. Factors are all of the numbers that will divide evenly into one number.

6.   A number is called a multiple of another number if it can be expressed as the product of that number and a second number. For example, the multiples of 4 are 4, 8, 12, 16, etc., because 4 × 1 = 4,4 × 2 = 8,4 × 3 = 12,4 × 4 = 16, etc.

7.   The Greatest Common Factor (GCF) is the largest integer that will divide evenly into any two or more integers. The Least Common Multiple (LCM) is the smallest integer into which any two or more integers will divide evenly. For example, the Greatest Common Factor of 24 and 36 is 12, because 12 is the largest integer that will divide evenly into both 24 and 36. The Least Common Multiple of 24 and 36 is 72, because 72 is the smallest integer into which both 24 and 36 will divide evenly.

8.   Multiplying and dividing both the numerator and the denominator of a fraction by the same nonzero number will result in an equivalent fraction.

9.   When multiplying fractions, multiply the numerators to get the numerator of the product, and multiply the denominators to get the denominator of the product. For example,

10.   To divide fractions, multiply the first fraction by the reciprocal of the second fraction. For example,  which equals

11.   When adding and subtracting like fractions, add or subtract the numerators and write the sum or difference over the denominator. So,

12.   When adding or subtracting unlike fractions, first find the Lowest Common Denominator. The Lowest Common Denominator is the smallest integer into which all of the denominators will divide evenly.

For example, to add  and , find the smallest integer into which both 4 and 6 will divide evenly. That integer is 12, so the Lowest Common Denominator is 12. Multiply  by  to get , and multiply  by  to get . Now add the fractions: , which can be simplified to .

13.   Place value refers to the value of a digit in a number relative to its position. Moving left from the decimal point, the values of the digits are 1’s, 10’s, 100’s, etc. Moving right from the decimal point, the values of the digits are 10ths, 100ths, 1000ths, etc.

14.   When converting a fraction to a decimal, divide the numerator by the denominator.

#### Square Roots

A square root is written , and is the nonnegative value a that fulfills the expression a2 = n. For example, the square root of 25 would be written as , which is equivalent to 52, or 5 × 5. A number is considered a perfect square when the square root of that number is a whole number. So, 25 is a perfect square because the square root of 25 is 5.

#### Exponents

When a whole number is multiplied by itself, the number of times it is multiplied is referred to as the exponent. As shown above with square roots, the exponent of 52 is 2 and it signifies 5 × 5. Any number can be raised to any exponential value. For example, 76 = 7 × 7 × 7 × 7 × 7 × 7 = 117,649.

#### Scientific Notation

When numbers are very large or very small, scientific notation is used to shorten them. To form the scientific notation of a number, the decimal point is moved until it is placed after the first nonzero digit from the left in the number. For example, 568,000,000 written in scientific notation would be 5.68 × 108, because the decimal point was moved 8 places to the left. Likewise, 0.0000000354 written in scientific notation would be 3.54 × 10−8, because the decimal point was moved 8 places to the right.

#### Ratio, Proportion, and Percent

A ratio is the relation between two quantities expressed as one divided by the other. For example, if there are 3 blue cars and 5 red cars, the ratio of blue cars to red cars is , or 3:5. A proportionindicates that one ratio is equal to another ratio. For example, if the ratio of blue cars to red cars is , and there are 8 total cars, you could set up a proportion to calculate the percent of blue cars, as follows:

3 cars is to 8 cars as x percent is to 100 percent

; solve for x

8x = 300

x = 37.5%

A percent is a fraction whose denominator is 100. The fraction  is equal to 55%.

#### Linear Equations with One Variable

In a linear equation with one variable, the variable cannot have an exponent or be in the denominator of a fraction. An example of a linear equation is 2x + 13 = 43. The ACT Mathematics Test will most likely require you to solve for x in that equation. Do this by isolating x on the left side of the equation, as follows:

One common ACT example of a linear equation with one variable is in questions involving speed of travel. The basic formula to remember is Rate × Time = Distance. The question will give you two of these values and you will have to solve for the remaining value.

#### Absolute Value

The absolute value of a number is notated by placing that number inside two vertical lines. For example, the absolute value of 10 is written as follows: |10|. Absolute value can be defined as the numerical value of a real number without regard to its sign. This means that the absolute value of 10, |10|, is the same as the absolute value of –10, |–10|, in that they both equal 10. Think of it as the distance from –10 to 0 on the number line and the distance from 0 to 10 on the number line: both distances equal 10 units.

#### Simple Probability

Probability is used to measure how likely an event is to occur. It is always between 0 and 1; an event that will definitely not occur has a probability of 0, whereas an event that will certainly occur has a probability of 1. To determine probability, divide the number of outcomes that fit the conditions of an event by the total number of outcomes. For example, the chance of getting heads when flipping a coin is 1 out of 2, or . There are two possible outcomes (heads or tails) but only one outcome (heads) that fits the conditions of the event. Therefore, the probability of the coin toss resulting in heads is 0.5, or 50%.

When two events are independent, meaning the outcome of one event does not affect the other, you can calculate the probability of both occurring by multiplying the probabilities of each of the events together. For example, the probability of flipping three heads in a row would be . The ACT Mathematics Test will assess your ability to calculate simple probabilities in everyday situations.

**Elementary Algebra**

The Elementary Algebra (eighth- or ninth-grade level) questions test elementary algebraic concepts such as:

1.      Functions

2.      Polynomial Operations and Factoring Simple Quadratic Expressions

3.      Linear Inequalities with One Variable

4.      Properties of Integer Exponents and Square Roots

#### Functions

A function is a set of ordered pairs where no two of the ordered pairs has the same x-value. In a function, each input (x-value) has exactly one output (y-value). An example of this relationship would be y = x2. Here, y is a function of x, because for any value of x there is exactly one value of y. However, x is not a function of y, because for certain values of y there is more than one value of x. The domain of a function refers to the x-values, while the range of a function refers to the y-values. If the values in the domain corresponded to more than one value in the range, the relation is not a function. The following is an example of a function question that may appear on the ACT Mathematics Test:

For the function f(x) = x2 − 3x, what is the value of f(5)?

Solve this problem by substituting 5 for x wherever x appears in the function:

f (x) = x2 − 3x

f (5) = (5)2 − (3)(5)

f (5) = 25 − 15

f (5) = 10

#### Polynomial Operations and Factoring Simple Quadratic Expressions

A polynomial is the sum or difference of expressions like 2x2 and 14x. The most common polynomial takes the form of a simple quadratic expression, such as 2x2 + 14x + 8, with the terms in decreasing order. The standard form of a simple quadratic expression is ax2 + bx + c, where a, b, and c are whole numbers. When the terms include both a number and a variable, such as x, the number is called the coefficient. For example, in the expression 2x, 2 is the coefficient of x.

The ACT Mathematics Test will often require you to evaluate, or solve a polynomial by substituting a given value for the variable, as follows:

For x = −2, 2x2 + 14x + 8 = ?

2(−2)2 + 14(−2) + 8

2(4) + (−28) + 8

8 − 28 + 8

= −12

You will also be required to add, subtract, multiply, and divide polynomials. To add or subtract polynomials, simply combine like terms, as in the following examples:

and

To multiply polynomials, use the distributive property to multiply each term of one polynomial by each term of the other polynomial. Following are some examples:

(3x)(x2 + 4x − 2) = (3x3 + 12x2 − 6x)

Remember the **FOIL** Method whenever you see this type of multiplication: multiply the **F**irst terms, then the **O**utside terms, then the **I**nside terms, then the Last terms.

(2x2 + 5x)(x − 3) =

**F**irst terms (2x2)(x) = 2x3

**O**utside terms: (2x2)(−3) = −6x2

**I**nside terms: (5x)(x) = 5x2

**L**ast terms: (5x)(−3) = −15x

Now put the terms in decreasing order:

2x3 + (−6x2) + 5x2 + (−15x) = 2x3 − 1x2 − 15x

You may also be asked to find the factors or solution sets of certain simple quadratic expressions. A factor or solution set takes the form (x ± some number). Simple quadratic expressions will usually have two of these factors or solution sets. Remember that the standard form of a simple quadratic expression is ax2 + bx + c. To factor the equation, find two numbers that when multiplied together will give you c and when added together will give you b.

The ACT Mathematics Test includes questions similar to the following:

What are the solution sets for x2 + 9x + 20?

Follow these steps to solve:

x2 + 9x + 20 = 0

(x + \_\_)(x + \_\_\_) = 0

5 and 4 are two numbers that when multiplied together give you 20, and when added together give you 9.

(x + 5)(x + 4) are the two solution sets for x2 + 9x + 20

#### Linear Inequalities with One Variable

Linear inequalities with one variable are solved in almost the same manner as linear equations with one variable: by isolating the variable on one side of the inequality. Remember, though, that when multiplying one side of an inequality by a negative number, the direction of the sign must be reversed.

The ACT Mathematics Test will include questions similar to those that follow:

For which values of x is 3x + 4 > 2x + 1?

Follow these steps to solve:

3x + 4 > 2x + 1

3x − 2x > 1 − 4

x > −3

For which values of x is 6x − 32 < 10x + 12?

Follow these steps to solve:

6x − 32 < 10x + 12

6x − 10x < 32 + 12

−4x < 44

Now, since you have to divide both sides by –4, remember to reverse the inequality sign: x > −11.

#### Properties of Integer Exponents

The ACT Mathematics Test will assess your ability to multiply and divide numbers with exponents. The following are the rules for operations involving exponents:

•

•

•

•

•    when x ≠ 0

•    when x ≠ 0

•    when x ≠ 0

**Intermediate Algebra**

The Intermediate Algebra (ninth- or tenth-grade level) questions test intermediate algebraic concepts such as:

1.      Quadratic Formula

2.      Radical and Rational Expressions

3.      Inequalities and Absolute Value Equations

4.      Sequences

5.      Systems of Equations

6.      Logarithms

7.      Roots of Polynomials

8.      Matrices

#### Quadratic Formula

The quadratic formula is expressed as  This formula finds solutions to quadratic equations of the form ax2 + bx + c = 0. It is the method that can be used in place of factoring for more complex polynomial expressions. The quantity b2 − 4ac is called the discriminant and can be used to determine quickly at what kind of answer you should arrive. If the discriminant is 0, then there is only one solution. If the discriminant is positive, then there are two real solutions. If the discriminant is negative, then you will have two complex solutions of the form (a + bi), where a and b are real numbers and i is the imaginary number defined by i2 = −1.

#### Radical and Rational Expressions

The nth root of a given quantity is indicated by the radical sign,  For example,  is considered a radical, and 9 is the radicand. The following rules apply to computations with radical signs:

•    means the “square root of a,”  means the “cube root of a,” etc.

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•

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A rational number is a number that can be expressed as a ratio of two integers. Fractions are rational numbers that represent a part of a whole number. To find the square root of a fraction, simply divide the square root of the numerator by the square root of the denominator. If the denominator is not a perfect square, rationalize the denominator by multiplying both the numerator and the denominator by a number that would make the denominator a perfect square.

Consider the following example:

#### Inequalities and Absolute Value Equations

An inequality with |ax + b|> c, an absolute value will be in the form of or |ax + b| < c, To solve |ax + b| > c, first drop the absolute value and create two separate inequalities with the word OR between them. To solve |ax + b| < c, first drop the absolute value and create two separate inequalities with the word AND between them. To remember this, think of the inequality sign that is being used in the equation. If it is a “greatOR” than sign, use OR. If it is a “less ‘thAND’ ” sign, use AND. The first inequality will look just like the original inequality without the absolute value. For the second inequality, you must switch the inequality sign and change the sign of c.

To solve |x + 3|> 5, first drop the absolute value sign and create two separate inequalities with the word OR between them:

x + 3 > 5 OR x + 3 < −5. Then solve for x:

x > 2 OR x < −8.

To solve |x + 3|< 5, first drop the absolute value sign and create two separate inequalities with the word AND between them:

x + 3 < 5 AND x + 3 > −5. Then solve for x:

x < 2 AND x > −8.

#### Sequences

An arithmetic sequence is one in which the difference between consecutive terms is the same. For example, 2, 4, 6, 8..., is an arithmetic sequence where 2 is the common difference. In an arithmetic sequence, the nth term can be found using the formula an = a1 + (n − 1)d, where d is the common difference.

A geometric sequence is one in which the ratio between two terms is constant. For example,  ... is a geometric sequence where 2 is the common ratio. With geometric sequences, you can find the nth term using the formula an = a1(r)n−1 where r is the common ratio.

#### Systems of Equations

The most common type of system of equations question tested on the ACT Mathematics Test involves two equations and two unknowns. Solve this system of equations as follows:

4x + 5y = 21

5x + 10y = 30

If you multiply the top equation by –2, you will get:

−8x − 10y = −42

Now, you can add the like terms of the two equations together, and solve for x:

(−8x + 5x) = −3x

(−10y + 10y) = 0

−42 + 30 = −12

−3x = −12

Notice that the two y-terms cancel each other out. Solving for x, you get . Now, choose one of the original two equations, plug 4 in for x, and solve for y:

4(4) + 5y = 21

16 + 5y = 21

5y = 5

y = 1

#### Logarithms

Logarithms are used to indicate exponents of certain numbers called bases, where logab = c, if ac = b. For example, log216 = 4, which means the log to the base 2 of 16 is 4, because 24 = 16.

The following is the kind of logarithm problem you are likely to see on the ACT Mathematics Test:

Which of the following is the value of x that satisfies logx9 = 2?

Follow these steps to solve:

logx9 = 2 means the log to the base x of 9 = 2.

So, x2 must equal 9, and x must equal 3.

#### Roots of Polynomials

When given a quadratic equation, ax2 + bx + c = 0, you may be asked to find the roots of the equation. This means you need to find what value(s) of x make the equation true. You may either choose to factor the quadratic equation or you may choose to use the quadratic formula. For example, use factoring to find the roots of x2 + 6x + 8 = 0 :

x2 + 6x + 8 = 0

(x + 4)(x + 2) = 0; solve for x

x + 4 = 0 and x + 2 = 0, so x = −4 and x = −2

The roots of x2 + 6x + 8 = 0 are x = −4 and x = −2. Using the quadratic formula will yield the same solution.

#### Matrices

A matrix is used to organize data in columns and rows. The dimensions of a matrix refer to the number of rows and columns of a given matrix. For example, a 2x3 matrix, as shown below, has two rows and three columns:

You can add and subtract matrices if each matrix has the same dimensions. Simply add or subtract the numbers that are in the same spot. For example:

You can also multiply matrices. When you multiply a matrix by a number, you multiply every element in the matrix by the same number. For example:

**Coordinate Geometry**

The Coordinate Geometry (Cartesian Coordinate Plane) questions test coordinate geometry concepts such as:

1.   Number Line Graphs

2.   Equation of a Line

3.   Slope

4.   Parallel and Perpendicular Lines

5.   Distance and Midpoint Formulas

#### Number Line Graphs

The most basic type of graphing is graphing on a number line. For the most part, you will be asked to graph inequalities. Below are four of the most common types of problems you will be asked to graph on the ACT Mathematics Test:

If the inequality sign specifies “greater than or equal to” (≥), or “less than or equal to” (≤), you would use a closed circle instead of an open circle on the designated number or the number line.

#### Equation of a Line

There are three forms used to write an equation of a line. The standard form of an equation of a line is in the form Ax + By = C. This can be transformed into the slope-intercept form of y = mx + b, where m is the slope of the line and b is the y-intercept (that is, the point at which the graph of the line crosses the y-axis). The third form is point-slope form, which is (y − y1) = m(x − x1), where m is the slope and (x1, y1) is a given point on the line. The ACT Mathematics Test will often require you to put the equation of a line into the slope-intercept form to determine either the slope or the y-intercept of a line as follows:

What is the slope of the line given by the equation 3x + 7y − 16 = 0?

Follow these steps to solve:

3x + 7y − 16 = 0; isolate y on the left side of the equation.

7y = −3x + 16

The slope of the line is

#### Slope

The slope of a line is the grade at which the line increases or decreases. Commonly defined as “rise over run,” the slope is a value that is calculated by taking the change in y-coordinates divided by the change in x-coordinates for any two given points on a line. The formula for slope is  where (x1, y1) and (x2, y2) are the two given points. For example, if you are given (3, 2) and (5, 6) as two points on a line, the slope would be  A positive slope means the graph of the line will go up and to the right. A negative slope means the graph of the line will go down and to the right. A horizontal line has slope 0, and a vertical line has undefined slope.

#### Parallel and Perpendicular Lines

Two lines are parallel if and only if they have the same slope. Two lines are perpendicular if and only if the slope of either of the lines is the negative reciprocal of the slope of the other line. To illustrate, if the slope of line a is 5, then the slope of line b must be  for lines a and b to be perpendicular.

#### Distance and Midpoint Formulas

To find the distance between two points on a coordinate graph, use the formula  where (x1, y1) and (x2, y2) are the two given points. For instance, the distance between (3, 2) and (7, 6) is

Note: This formula is based on the Pythagorean Theorem and if you can’t remember it on test day, just draw a right triangle on your test booklet and proceed from there.

To find the midpoint of a line given two points on the line, use the formula . For example, the midpoint between (5, 4) and (9, 2) is

**Plane Geometry**

Plane Geometry questions test plane geometry concepts such as:

1.   Properties and Relations of Plane Figures

a.   Triangles

b.   Circles

c.   Rectangles

d.   Parallelograms

e.   Trapezoids

2.   Angles, Parallel Lines, and Perpendicular Lines

3.   Perimeter, Area, and Volume

#### Properties and Relations of Plane Figures

**Triangles**

A triangle is a polygon with three sides and three angles. If the measure of all three angles in the triangle are the same and all three sides of the triangle are the same length, then the triangle is an equilateral triangle. If the measure of two of the angles and two of the sides of the triangle are the same, then the triangle is an isosceles triangle.

The sum of the interior angles in a triangle is always 180°. If the measure of one of the angles in the triangle is 90° (a right angle), then the triangle is a right triangle, as shown below.

Some right triangles have unique relationships between the angles and the lengths of the sides. These are called special right triangles. It may be helpful to remember the following information:

The perimeter of a triangle is the sum of the lengths of the sides. The area of a triangle is  (base)(height). For any right triangle, the Pythagorean Theorem states that a2 + b2 = c2, where aand b are legs (sides) and c is the hypotenuse.

##### Circles

The equation of a circle centered at the point (h, k) is (x − h)2 + (y − k)2 = r2, where r is the radius of the circle. The radius of a circle is the distance from the center of the circle to any point on the circle. The diameter of a circle is twice the radius. The formula for the circumference of a circle is C = 2πr, and the formula for the area of a circle is A = πr2.

##### Rectangles

A rectangle is a polygon with two pairs of congruent, parallel sides and four right angles. The sum of the angles in a rectangle is always 360°. The perimeter of a rectangle is P = 2l + 2w, where l is the length and w is the width. The area of a rectangle is A = lw. The lengths of the diagonals of a rectangle are congruent, or equal. A square is a special rectangle where all four sides are of equal length, as shown here:

##### Parallelograms

A parallelogram is a polygon with four sides and four angles that are NOT right angles. A parallelogram has two sets of congruent sides and two sets of congruent angles.

Again, the sum of the angles of a parallelogram is 360°. The perimeter of a parallelogram is P = 2l + 2w. The area of a parallelogram is A = (base)(height). The height is the distance from top to bottom. A rhombus is a special parallelogram with four congruent sides.

##### Trapezoids

A trapezoid is a polygon with four sides and four angles. The bases of the trapezoid (top and bottom) are never the same length. The sides of the trapezoid can be the same length (isosceles trapezoid), or they may not be. The perimeter of the trapezoid is the sum of the lengths of the sides. The area of a trapezoid is  (Base1 + Base2) (Height). Height is the distance between the bases. (The diagonals of an isosceles trapezoid have a unique feature. When the diagonals of a trapezoid intersect, the ratio of the top of the diagonals to the bottom of the diagonals is the same as the ratio of the top base to the bottom base.)

#### Angles, Parallel Lines, and Perpendicular Lines

Angles can be classified as acute, obtuse, or right. An acute angle is any angle less than 90°. An obtuse angle is any angle that is greater than 90° and less than 180°. A right angle is an angle that is 90°.

When two parallel lines are cut by a perpendicular line, right angles are created, as follows:

When two parallel lines are cut by a transversal, the angles created have special properties. Each of the parallel lines cut by the transversal has four angles surrounding the intersection that are matched in measure and position with a counterpart at the other parallel line. The vertical (opposite) angles are congruent, and the adjacent angles are supplementary; that is, the sum of the two supplementary angles is 180°.

**Note:** Almost every ACT ever administered has a diagram similar to the one above as part of at least one math question.

#### Perimeter, Area, and Volume

These formulas are not provided for you on test day. You should make your best effort to memorize them.

##### Perimeter

The formulas for calculating the perimeter of shapes that appear on the ACT Mathematics Test are as follows:

Triangle: Sum of the Sides

Rectangle and Parallelogram: 2l + 2w

Square: 4s (s is Length of Side)

Trapezoid: Sum of the Sides

Circle (Circumference): 2r

##### Area

The formulas for calculating the area of shapes that appear on the ACT Mathematics Test are as follows:

Triangle:  (Base)(Height)

Rectangle and Square: (Length)(Width)

Parallelogram: (Base)(Height)

Trapezoid:  (Base1 + Base2)(Height)

Circle: πr2

##### Volume

The formulas for calculating the volume of basic three-dimensional shapes that appear on the ACT Mathematics Test are as follows:

Rectangular Box and Cube: (Length)(Width)(Height)

Sphere:

Right Circular Cylinder: πr2h (h is the height)

Right Circular Cone:  (h is the height)

Prism: (Area of the Base)(Height)

**Trigonometry**

There are typically only four trigonometry questions on the ACT Mathematics Test. If you have never taken trigonometry in school, you may still be able to learn enough here to get by on at least a couple of the four questions. (Even if you NEVER learn trigonometry, don’t worry; four questions are not likely to seriously affect your score.) The questions test the basic trigonometric ratios (which are related to right triangles, as shown below).

#### Basic Trigonometric Concepts

The hypotenuse is the side that is opposite the right angle. Sometimes the graph or diagram shown in the question will have the triangle rotated, so make sure that you know where the right angle is and, of course, the hypotenuse, which is directly opposite the right angle.

SOHCAHTOA

**S**INE (sin) = **O**pposite/**H**ypotenuse (SOH)

**C**OSINE (cos) = **A**djacent/**H**ypotenuse (CAH)

**T**ANGENT (tan) = **O**pposite/**A**djacent (TOA)

#### Advanced Trigonometric Concepts

Note: The following information will be extremely confusing and intimidating for anyone who has never heard of it before. This information is included only as a review for those readers who have had a trigonometry class. The rest of you should just guess on the two or three questions that might include these concepts.

The secant, cosecant, and cotangent can be found as follows:

Remember the following Pythagorean Identities:

sin2 θ + cos2 θ = 1

1 + tan2 θ = sec2 θ

1 + cot2 θ = csc2 θ

Remember the following Trigonometric Identities:

#### Radians

To change from degrees to radians, multiply the number of degrees by  For example, 120° is  radians. Conversely, to change from radians to degrees, multiply the number of radians by

### ACT MATHEMATICS SKILLS EXERCISES

The next few pages contain exercises designed to help you apply the concepts generally tested on the ACT Mathematics Test. Following this section are simulated ACT Mathematics questions, which will allow you to become familiar with the format and types of questions you’ll see on your actual ACT test. You might want to get some scratch paper before starting this section.

**Basic Operations**

These questions will test your knowledge of operations using whole numbers, fractions, and decimals.

**Insert the correct operator in the blanks below.**

1.   108 \_\_ 9 = 12

2.   7 \_\_ 2 = 3.5

3.

**Answer the following questions.**

4.   What is the greatest common factor of 48 and 72?

5.   What is the lowest common denominator of  and

**Solve the following equations.**

6.

7.   3(27 + 2 - 3) = \_\_\_\_\_

8.

9.   231.2 − 198.7 = \_\_\_\_\_

10.

**Exponents and Square Roots**

These questions will test your knowledge of operations using square roots.

**Solve the following problems.**

1.   52 = \_\_\_\_\_

2.

3.   Express 3 × 3 as a square: \_\_\_\_\_

4.   72 − 32 = \_\_\_\_\_

5.

**Properties of Integer Exponents**

These questions will test your knowledge of operations involving integer exponents.

**Solve the following problems.**

1.   x3 × x6 = \_\_\_\_\_

2.   (32)3 = \_\_\_\_\_

3.

4.   137° = \_\_\_\_\_

5.   (y × z)2 = \_\_\_\_\_

**Fill in the blanks below with the correct number.**

1.   2 raised to the power of \_\_\_\_\_ = 8.

2.   33 = \_\_\_\_\_

3.   \_\_\_\_\_4 = 81

4.   125 = 5 \_\_\_\_\_

5.   (24)2 = \_\_\_\_\_

**Scientific Notation**

These questions will test your knowledge of operations using scientific notation.

**Fill in the blanks below with the correct number.**

1.   423,700,000 = 4.237 × 10 to the power of \_\_\_\_\_

2.   3.76 × 105 = \_\_\_\_\_

3.   (2.50 × 104) ÷ (1.25 × 103) = \_\_\_\_\_

4.   6.47 − 10−5 = \_\_\_\_\_

5.   (4.2 × 103) × (1.8 × 10−6) = \_\_\_\_\_

**Ratio, Proportion, and Percent**

These questions will test your knowledge of operations involving ratio, proportion, and percent.

**Answer the following questions.**

1.   \_\_\_\_ is 30% of 20.

2.    Solve for x.

3.   As an assistant analyst for the Department of Natural Resources, you were asked to analyze samples of river water. A 2-liter sample of water contained about 24 of a particular organism and a 4-liter sample of water contained about 48 such organisms. At this rate, how many of the organisms would you expect to find in a 10-liter sample of water from the same river? \_\_\_\_\_

4.   If 20% of x equals 16, then x = \_\_\_.

5.   Jim scored 95 points in 5 basketball games for his school. At this rate, how many points will he have scored by the end of the 12-game season?

**Linear Equations with One Variable**

These questions will test your knowledge of linear equations involving one variable.

**Solve the following equations.**

1.   3x − 17 = 46. Solve for x.

2.    Solve for x.

3.   If x = 15, then 4x − \_\_\_\_ = 42.

4.   Two trains running on parallel tracks are 600 miles apart. One train is moving east at a speed of 90 mph, while the other is moving west at 75 mph. How long will it take for the two trains to pass each other?

5.   3(x − 4) = 5x − 20. Solve for x.

**Absolute Value**

These questions will test your knowledge of operations involving absolute value.

**Solve the following equations.**

1.   If x = −8, what is the value of |x − 6|?

2.   Solve |4x − 6| = 10 for x.

3.   |−15| × |6| = \_\_\_\_

4.   Solve |6x + 8| = |3x − 7| for x.

5.

**Simple Probability**

These questions will test your knowledge of operations involving simple probability.

**Answer the following questions.**

1.   If you roll a single 6-sided die, what is the probability that you will roll an odd number?

2.   A company knows that 2.5% of the CD players it makes are defective. If the company produces 300,000 CD players, how many will be defective?

3.   When flipping a coin, what is the probability that it will land on tails four times in a row?

4.   If the probability that Dave will go to class is 0.7, what is the probability that he will not go to class?

5.   There is a bowl with 20 marbles in it (8 blue, 6 red, 3 green, 2 yellow, and 1 orange.) If you reach in and choose one marble at random, what is the probability that it will be red?

**Functions**

These questions will test your knowledge of operations involving functions.

**Answer the following questions.**

1.   For the function f(x) = x2 − 4x + 8, what is the value of f(6)?

2.   If f(x) = x2, find f(x + 1).

3.   If the function f(x) = x + 2, and the function g(x) = 3x, what is the function g(f(x))?

4.   For the function  what is the value of f(2)?

5.   For the function f(x) = x2 + x, what is the value of f(−5)?

**Polynomial Operations and Factoring Simple Quadratic Equations**

These questions will test your knowledge of operations involving polynomial operations and factoring simple quadratic equations.

**Solve the following equations.**

1.   For x = 4, 3x2 − 5x + 9 = \_\_\_\_

2.   (5x3 + 3x − 12) − (2x3 − 6x + 17) = \_\_\_\_\_

3.   (4x2 + 2x)(x − 6) = \_\_\_\_\_

**Answer the following questions.**

4.   What are the solution sets for x2 + 2x − 48?

5.   (x − 4) and (2x + 3) are the solution sets for what equation?

**Linear Inequalities with One Variable**

These questions will test your knowledge of operations involving linear inequalities with one variable.

**Answer the following questions.**

1.   For −5 ≤ x < 15, x = \_\_\_\_

2.   For which values of x is 6x − 3 > 4x + 5?

3.   If x = 7, the is 3x + 7 greater than or less than 5x − 6?

4.   For which values of x is 2x − 5 < −3x + 20?

5.   Solve −4 ≤ x + 3 < 18 for x.

**Quadratic Formula**

These questions will test your knowledge of operations involving the quadratic formula.

**Answer the following questions.**

1.   Use the quadratic formula to solve the equation 10x2 + 22x + 12.1 = 0.

2.   Set up the equation 4x2 − 7x + 3 = 10x2 + x − 11 so it can be used in the quadratic formula.

3.   Solve the equation 4x2 + x − 5 = 0 using the quadratic formula.

4.   Which values of a, b, and c will you use in the quadratic formula for the equation 18x − 117 + 4x2 = 0? (Place an “X” next to the correct answer.)

\_\_\_18,−117,4

\_\_\_−117,4,18

\_\_\_ 4,18,−117

\_\_\_ 4,18,117

5.   Solve the equation (2x + 4)2 = 0 using the quadratic formula.

**Radical and Rational Expressions**

These questions will test your knowledge of operations involving radical and rational expressions.

**Solve the following problems.**

1.

2.

3.

4.

5.

**Inequalities and Absolute Value Equations**

These questions will test your knowledge of operations involving inequalities and absolute value equations.

**Answer the following questions.**

1.   For |7x − 13|< 22, which one of the following is true? (Place an “X” next to the correct answer.)

\_\_\_ 7x − 13 > 22 OR 7x − 13 < −22

\_\_\_ 7x −13 < 22 AND 7x −13 > −22

\_\_\_− 7x −13 < 22 AND − 7x −13 > −22

\_\_\_ 7x +13 > 22 OR 7x + 13 < −22

2.   If |x + 8|> 15, what is/are the possible values of x?

3.   If |2x + 3|< 21, what is/are the possible values of x?

4.   For |5x − 6|> 29, which one of the following is true? (Place an “X” next to the correct answer.)

\_\_\_ 5x − 6 > 29 OR 5x − 6 < −29

\_\_\_ 5x − 6 < 29 AND 5x − 6 > −29

\_\_\_− 5x − 6 < 29 AND − 5x − 6 > −29

\_\_\_ 5x + 6 > 29 OR 5x + 6 < −29

5.   If  what is/are the possible values of x?

**Sequences**

These questions will test your knowledge of operations involving sequences.

**Answer the following questions.**

1.   Find the 3rd term of the arithmetic sequence: an = 3 + (n − 1)(2).

2.   Write a formula for the nth term of the arithmetic sequence –8, –2, 4, 10,...

3.   In the geometric sequence:  what is the 6th term?

4.   Which of the following represents the formula to find the 8th term of the arithmetic sequence 7, 13, 19, 25,...? (Place an “X” next to the correct answer.)

\_\_\_13 + (8 − 1)(19)

\_\_\_ 25(7)19−13

\_\_\_ 7(6)8−1

\_\_\_ 7 + (8 − 1)(6)

5.   Write a formula for the nth term of the geometric sequence 25, –5, 1,

**Systems of Equations**

These questions will test your knowledge of operations involving systems of equations.

**Solve the following systems of equations.**

1.

2.

3.

4.

5.

**Logarithms**

These questions will test your knowledge of operations involving logarithms.

**Solve the following problems.**

1.   What is the value of x that satisfies logx27 = 3?

2.   If logx625 = 4, what is the value of x? (Place an “X” next to the correct answer.)

\_\_\_ 4

\_\_\_ 5

\_\_\_ 7

\_\_\_ 19

3.   log3729 =?

4.   If logx196 = 2, then x =\_\_\_?

5.   If log7x = 3, what is the value of x? (Place an “X” next to the correct answer.)

\_\_\_ 5

\_\_\_ 64

\_\_\_ 216

\_\_\_ 343

**Roots of Polynomials**

These questions will test your knowledge of operations involving roots of polynomials.

**Answer the following questions.**

1.   Find the roots of 2x2 + 9x − 35 by factoring.

2.   Find the roots of x2 + 2x − 3 by factoring.

3.   What polynomial equation has the solutions x = 6 and x = −2?

4.   Solve for x by factoring the polynomial equation x2 − 8x + 16.

5.   Find the roots of −x2 + 3x + 40 by factoring.

**Number Line Graphs**

These questions will test your knowledge of operations involving number line graphs.

**Answer the following questions.**

1.   On a number line, what is the distance between –5 and 3?

2.   What is the midpoint of the two points in the below graph?

3.   The below graph represents which values for x? (Place an “X” next to the correct answer.)

\_\_\_ x > −3 AND x < 6

\_\_\_ x ≥ −3 AND x ≤ 6

\_\_\_ x ≥ −3 OR x < 6

\_\_\_ x ≥ −3 AND x < 6

4.   The below graph represents the solution to which inequality? (Place an “X” next to the correct answer.)

\_\_\_ |2x − 10| < 6

\_\_\_ |2x + 10| < 6

\_\_\_ |2x − 10| > 6

\_\_\_ |− 2x + 10| > 6

5.   The below graph represents which values for x? (Place an “X” next to the correct answer.)

\_\_\_ x ≥ 2 OR x < −6

\_\_\_ x ≥ 2 AND x < −6

\_\_\_ x ≥ − 6 OR x < 2

\_\_\_ x ≥ − 6AND x > 2

**Equation of a Line and Slope of a Line**

These questions will test your knowledge of operations involving the equation of a line and the slope.

**Answer the following questions.**

1.   What is the y-intercept of the line with the equation 2y = 4x + 12?

2.   What is the slope of the line with the equation 3y = −2x + 5?

3.   What is the slope of the line x = 4?

4.   What is the equation of a line parallel to y = 4x − 12 and crossing the y-axis at 3?

5.   What is the equation of a line perpendicular to 3x = 2 − y with the y-intercept 8?

**Distance and Midpoint Formulas**

These questions will test your knowledge of operations involving distance and midpoint formulas.

**Answer the following questions.**

1.   What is the distance between the points (3, –4) and (9, 4)?

2.   What is one possible value for y if the distance between the two points (2, 8) and (–6, y) is 17?

3.   What is the midpoint between (12, 5) and (10, –7)?

4.   Solve for x if the midpoint between the two points (x, 1) and (–2, –3) is (5, –1).

5.   What is the distance between the points (0, 5) and (5, 0)?

**Properties and Relations of Plane Figures**

These questions will test your knowledge of operations involving plane figures.

**Answer the following questions.**

1.   What is the hypotenuse of a right triangle with a base of 9 cm and an area of 54 cm2?

2.   What is the area of a circle with a circumference of 14π inches?

3.   If one of the angles of a parallelogram measures 35°, what is the sum of the remaining angles?

4.   A trapezoid has one base of 8 ft, a height of 3 ft, and an area of 30 ft2, what is the length of the other base?

5.   A polygon with four sides and four right angles has one side of 6 mm. If the area is 42 mm2, would the polygon be considered a square or a rectangle?

**Angles, Parallel Lines, and Perpendicular Lines**

These questions will test your knowledge of operations involving angles, parallel lines, and perpendicular lines.

**Answer the following questions.**

1.   What is the measure of the angle that is supplementary to a 40° angle?

2.   What is the measure of the angle that is supplementary to a 25° angle?

3.   In the figure below, line n is parallel to line m, and line p is parallel to line o. What is the measure of angle θ?

4.   In the figure below, line x is parallel to line y. What is the measure of angle a?

5.   In the figure below, line t is parallel to line u, and line v is perpendicular to line u. What is the measure of angle a?

**Perimeter, Area, and Volume**

These questions will test your knowledge of operations involving perimeter, area, and volume.

**Answer the following questions.**

1.   You are applying fertilizer to your backyard. The rectangular yard measures 40 feet wide and 70 feet long. You use 6 pounds of fertilizer to treat 700 square feet. The fertilizer comes in 8-pound bags. How many bags of fertilizer will you need to complete the job?

2.   John is building a circular fence around his circular pool. The pool is 26 feet in diameter. If John wants to have 4 feet of space between the edge of the pool and the fence, what is the area that will be enclosed by the fence? (π = 3.14)

3.   Tiffany inflates a beach ball. If the diameter of the ball is 0.6 m, what is the volume?

4.   A cylindrical can of pineapple juice contains 350 cm3 of liquid. If the can is  cm tall, what is the diameter?

5.   A cube has an edge length of 5 in; what is the volume of the cube?

**Trigonometry**

These questions will test your knowledge of operations involving trigonometry.

**Answer the following questions.**

1.   In the triangle below, what is sin a?

2.   If cos  what is tan a?

3.   Convert 60° into radians.

4.   Convert  radians into degrees.

5.   If sec , what is sin a?

**Translating Word Problems**

These questions will test your ability to locate relevant mathematical information in word problems.

**Place an “X” next to the correct expression in the questions below.**

1.   Tom had 6 books. He gave 2 to his sister and then purchased 3 more at the bookstore. Which of the following mathematical expressions is equivalent to the number of books that Tom has now?

\_\_\_ 6 − 2 + 3

\_\_\_ 6 + 2 − 3

\_\_\_ 6(2 + 3)

\_\_\_ 6(2 − 3)

2.   Juan walked 3 more miles than Rebecca. Rebecca walked 4 times as far as William. William walked 2 miles. Which of the following mathematical expressions is equivalent to the number of miles Juan walked?

\_\_\_ 3 × 4 × 2

\_\_\_(2 + 4) × 3

\_\_\_ 4(2) + 3

\_\_\_ 4 + 3 + 2

3.   Tina goes to the store to purchase some CDs and DVDs. CDs cost $15 and DVDs cost $18. Which of the following expressions gives the total amount of money, in dollars, Tina will pay for purchasing 2 of the CDs and d of the DVDs?

\_\_\_15 + d

\_\_\_ 30 + 18d

\_\_\_18 + d + 30

\_\_\_ d(18 + 15)

4.   Mark is older than Frank, but younger than David. If m, f, and d represent the ages, in years, of Mark, Frank, and David, respectively, which of the following is true?

\_\_\_ d < f < m

\_\_\_ f < m < d

\_\_\_ d < m < f

\_\_\_ f < d < m

5.   Kathy was twice as old as Jim 2 years ago. Today, Jim is j years old. In terms of j, how old was Kathy 2 years ago?

\_\_\_ 2(j − 2)

\_\_\_ 2j − 2

\_\_\_ 2(j + 2)

\_\_\_ j(2 + 2)

### ANSWERS AND EXPLANATIONS

**Basic Operations**

1.   In order for 12 to be the result of this equation, you must divide 108 by 9. Insert the ÷ symbol in the blank.

2.   To reach an answer of 3.5, you must divide 7 by 2. Insert the ÷ symbol in the blank.

3.   One way to solve this problem is to look for the Lowest Common Denominator (LCD). The smallest number that both 4 and 8 divide evenly into is 8, so the fraction  does not need to be changed. The fraction  is equivalent to ,  plus  equals , so insert the + symbol in the blank.

4.   The Greatest Common Factor (GCF) is the largest number that divides evenly into any two or more numbers. List the factors of 48 and 72, then select the largest factor that they have in common:

Based on this list, the GCF is 24.

5.   The LCD is the smallest number into which all of the denominators will divide evenly. For this problem, you must find the smallest number into which 8 and 4 will divide evenly. Since 4 will divide evenly into  is your LCD. You can now change  to  by multiplying both the numerator and denominator by 2 (the amount of times 4 goes into 8).

6.   You must first complete the mathematics within the parentheses (96 − 21 = 75). Next, do any multiplication or division in the problem, from left to right. Here, you have 75 divided by 15, which equals 5. Finally, do any addition or subtraction in the problem, from left to right: 5 plus 11 gives us an answer of 16.

7.   You must first do the operations within the parentheses (27 + 2 − 3 = 26). Now multiply the value from the parentheses by 3: 3 times 26 = 78.

8.   You must first find the LCD for the two fractions involved. The denominators are 3 and 7. The smallest number into which both of these can divide evenly is 21. Convert each denominator to 21 by multiplying  by  and  by  This gives you  which equals .

9.   This is a simple subtraction problem. To solve this without a calculator, line up the decimal points and subtract, remembering to “borrow” and “carry,” as follows:

10.   First convert  to a decimal, which is 0.2. Then multiply 0.25 by 0.2, which gives you an answer of 0.05. Another way to solve this is to first convert 0.25 to a fraction, which is . When multiplying the two fractions, you first multiply the numerators, and then the denominators, giving you  Because this is equivalent to 0.05, either answer will be correct.

**Square Roots**

1.   52 simply means 5 times 5, which equals 25.

2.   Find the square roots before you do the division. The square root of 36 is 6, and the square root of 4 is 2. Next divide 6 by 2, which equals 3.

3.   “3 times 3” can be stated as “3 squared.” The proper way to write this is 32.

4.   Both numbers are raised to the power of 2 (they are squared). You must first find these squares before you do your subtraction. 7 squared is 49, and 3 squared is 9. So, your answer is 49 – 9, which equals 40.

5.   This problem requires you to find a square root of a number as well as a number squared. The square root of 64 is 8, and 2 squared equals 4. Your answer is 8 times 4, which is 32.

**Properties of Integer Exponents**

1.   According to the rule xm × xn = x(m−n); therefore, add the exponents together. x3 × x6 is equal to x3+6, or x3+6 or x9.

2.   A rule regarding exponents states that (xm)n = xmn. Applying this rule gives you (32)3, which yields 36 . 3 to the 6th power is 729.

3.   The exponent is distributed to both the numerator and the denominator, creating  or

4.   The answer to this problem is 1. For any value x where x ≠ 0, x0 = 1.

5.   One of the rules regarding exponents tells you that (xy)m = xm × ym. Applying the rule gives you the following:

y2 × z2, or y2 z2

**Exponents**

1.   The power that a number is raised to is equivalent to the number of times you multiply that number by itself: 2 × 2 × 2 × is equal to 8, so the answer is 2 raised to the power of 3 (23).

2.   33, or 3 to the 3rd power, means you must multiply 3 × 3 × 3 × which equals 27.

3.   You must find a number that, when raised to the power of 4, equals 81. Because 81 is a perfect square (9 × 9, or 92 = 81), and 9 is a perfect square, (3 × 3, or 32 = 9), you can simply square 32to arrive at 81: (32)2 = 34.

4.   53 = 5 × 5 × 5,, which gives you 125.

5.   When raising an exponent to another power, multiply the exponents (4 × 2 = 8). So, the answer is 28, or 256.

**Scientific Notation**

1.   When dealing with scientific notation, the power of 10 indicates the number of spaces you must move the decimal place, either to the right (for a positive value), or to the left (for a negative value). To turn 4.237 into 423,700,000, you must move the decimal place 8 spaces to the right. Therefore, 10 needs to be raised to the power of 8 (108).

2.   To solve this problem, you must simply move the decimal point the number of times indicated by the power of 10. Since you are given 105, you know that you must move the decimal point 5 spaces to the right because the exponent is a positive number. This gives you an answer of 376,000.

3.   This problem can be set up as  The first half  gives you 2. When dividing like bases, you subtract your exponents (4 − 3 = 1). You are left with 2 × 101. Since 10 to the 1st power is 10, the multiplication leaves you with an answer of 20.

4.   You are given a negative value for the power to which 10 is raised (–5). This means that you must move the decimal point 5 spaces to the left to get your answer, which is .0000647.

5.   You can set this problem up as (4.2 × 1.8) × (103 × 10−6). The first half of the equation (4.2 × 1.8) gives you 7.56. When multiplying like bases, you add your exponents: 3 + (−6) = −3. Therefore, you are left with 7.56 × 10−3, which can be expressed as 0.00756.

**Ratio, Proportion, and Percent**

1.   To solve this problem, you can set up a proportion. You are looking for a number that is 30% of 20. The proportion looks like  because the unknown number is equivalent to 30 out of the 100 parts of the whole (20). To solve, you cross-multiply, leaving you with 100x = 600. Divide both sides by 100 : x = 6.

2.   You are given a proportion to solve. To find the answer, cross-multiply, giving you 78x = 234. Dividing both sides by 78 will give you the answer x = 3.

3.   To answer this question you must determine the ratio of organisms to liter of river water. The problem states that a 2-liter sample of water contained about 24 organisms, and a 4-liter sample of water contained about 48 organisms. Upon closer examination of this information you will see that the ratio of organism to water is the same in each sample. Therefore, you can set up a ratio using one sample:

2 liters of water yields 24 organisms.

This can be expressed as 2 to 24, or 2:24, which can be reduced to 1:12. For every 1 liter of water you will see 12 organisms. Therefore, 10 liters of water will contain 120 organisms.

4.   You need to set up a proportion. You are given that 20% of x is equal to 16, and you want to find the value of x. The proportion looked like this:

After cross-multiplying, you are left with 20x = 1,600. After dividing both sides by 20, you have the answer: x = 80.

5.   Once again, you need to use a proportion to solve this problem. You know that Jim scored 95 points in 5 games, and you want to find out how many points he will score in a total of 12 games. Your proportion will look like this:

Cross-multiplying will leave you with 5x = 1,140. Divide both sides by 5, and you get your answer, x = 228. If Jim continues to score at this rate, he will score a total of 228 points by the end of the season (12 games).

**Linear Equations with One Variable**

1.   First isolate the unknown number (the variable) on one side. To do this, you add 17 to both sides, giving you 3x = 63. Next, you divide both sides by 3 to get the x alone. This gives you the answer: x = 21.

2.   Multiply both sides by 4 to get rid of the fraction and leave the x on its own. This gives you x = −24.

3.   You are given the value of x, and you are looking for a missing number in the equation. If x = 15, then 4x = 60. So you are left with the equation 60– (some number) = 42. Subtract 60 from both sides to get 18.

4.   This is a standard Rate × Time = Distance problem. Since the two trains start 600 miles apart, you know that their combined distance traveled must equal 600. Using the R × T = D formula, you can say that (Rate of Train 1 × Time of Train 1) + (Rate of Train 2 × Time of Train 2) = 600. You know how fast the trains are moving, and their total distance, but you do not know the time, so solve for T. Train 1 travels at 90 mph for T hours, while Train 2 travels at 75 mph for T hours. Your equation will look like this:

90T + 75T = 600

165T = 600

T = 3.64 hours

5.   First do the multiplication on the left side of the equation. This gives you 3x − 12 = 5x − 20. Next, you need to group the like terms together. To do this, subtract 3x from both sides, and add 20 to both sides. This leaves you with 8 = 2x. Dividing both sides by 2 will give you the answer: x = 4.

**Absolute Value**

1.   First do the subtraction within the absolute value lines, (−8 − 6 = −14). Absolute value is the numerical value of a real number without regard to its sign. Therefore, the absolute value of –14 is 14.

2.   To solve this problem, you need to set up two equations:

4x − 6 = 10, and 4x − 6 = −10. You then solve both for x.

4x = 16, and 4x = −4

x = 4, and x = −1

3.   In order to perform the multiplication in this problem, you must first find the absolute value of both numbers. The absolute values of –15 and 6 are 15 and 6, respectively. The answer is 15 × 6, which equals 90.

4.   To find the possible answers for x in this problem, you must set up two equations:

6x + 8 = 3x − 7, and 6x + 8 = −(3x − 7)

First, you need to distribute the minus sign in the second equation, giving you 6x + 8 = −3x + 7.

You then solve both for x:

3x = −15, and 9x = −1

x = −5, and

5.   First find the absolute value of the denominator. The absolute value of –8 is 8. Now you can perform the division. –32 divided by 8 gives you an answer of –4.

**Simple Probability**

1.   On a 6-sided die, there are 3 even and 3 odd numbers. Therefore, the probability that you will roll an odd number is 3 out of 6, or . This can be reduced to  or .5

2.   If 2.5% of the CD players produced by this company are defective, then the number of defective devices out of 300,000 can be determined by multiplication 0.025 × 300,000 = 7,500.

3.   When flipping a coin, there are only two possible outcomes: heads or tails. Therefore, each side has a probability of  or .5, of landing facing up. The chances of the coin landing on tails four times in a row can be expressed as  or  The final answer is

4.   In this question, you can look at probability as a percentage. The probability that Dave will go to class is 0.7, or 70%. Therefore, the probability that he will NOT go to class is 100% – 70%, or 30%, which is equivalent to 0.3. Either answer is correct.

5.   There are a total of 20 marbles in the bowl, 6 of which are red. If one marble is selected at random, the probability that it will be red is  (the # of red marbles/the total # of marbles). This can be reduced to

**Functions**

1.   To solve, substitute 6 for x in the function:

f (6) = 62 − 4(6) + 8

f (6) = 36 − 24 + 8

f (6) = 20

2.   To solve, substitute (x + 1) for x in the function:

f (x + 1) = (x + 1)2

(x + 1)(x + 1)

x2 + x + x + 1

x2 + 2x + 1

3.   The problem gives g(x) = 3x and f(x) = x + 2 and asks for g(f(x)).

The function g(f(x)) means that all of the x values in g(x) are replaced with f(x), as follows:

g(f(x)) = 3(f(x))

g(f(x)) = 3(x + 2)

g(f(x)) = 3x + 6

4.   To solve, substitute 2 for x in the function:

f(2) = 16 − 3

f(2) = 13

5.   To solve, substitute –5 for x in the function:

f(−5) = (−5)2 + (−5)

f(−5) = 25 − 5

f(−5) = 20

**Polynomial Operations and Factoring Simple Quadratic Equations**

1.   To solve the equation, substitute 4 for x:

3(42) − 5(4) + 9

3(16) − 20 + 9

48 − 20 + 9 = 37

2.   To add or subtract polynomials, combine like terms (remember to keep track of the negative signs!):

(5x3 + 3x − 12) − (2x3 − 6x + 17)

(5x3 − 2x3) + (3x + 6x) − (17 − 12)

3x3 + 9x − 29

3.   Use the distributive property to multiply each term of one polynomial by each term of the other (remember to use the FOIL method).

(4x2 + 2x)(x − 6)

First terms: (4x2)(x) = 4x3

Outside terms: (4x2)(−6) = −24x2

Inside terms: (2x)(x) = 2x2

Last terms: (2x)(−6) = −12x

Now place the terms in decreasing order:

4x3 − 24x2 + 2x2 − 12x

4x3 − 22x2 − 12x

4.   Find two numbers whose product is –48 and sum is 2. The only possible numbers are 8 and –6. Therefore, the solution sets are (x – 6) and (x + 8).

5.   The solution sets are given, so multiply the two sets together to find the original equation, using the FOIL method:

(x − 4)(2x + 3)

2x2 + 3x − 8x − 12

2x2 − 5x − 12

**Linear Inequalities with One Variable**

1.   The inequality states that x must be greater than or equal to –5 AND less than 15. Therefore, x could be any number equal to or greater than –5, and also less than –15.

2.   Solve this problem algebraically, as follows:

6x − 4x > 5 − (−3)

2x > 8

x > 4

x must be greater than 4 for this inequality to be true.

3.   The value of x is given, so substitute 7 for x and calculate the value of both sides:

3(7) + 7 = 28 and 5(7) − 6 = 29

The less than sign (<) is used because 28 is less than 29.

4.   Once again, the first step in solving this problem is isolating the variable on one side of the inequality:

−5 − 20 < −3x − 2x

−25 < −5x

5 > x

It is important to remember that when dealing with inequalities, multiplying or dividing by a negative number involves reversing the sign. In this case, both sides were divided by –5, so the sign changes from < to >.

5.   To solve this problem, subtract 3 from both sides of the inequality and isolate x:

−4 − 3 ≤ x < 18 − 3

−7 ≤ x < 15

x is greater than or equal to –7 and it is less than 15.

**Quadratic Formula**

1.   The quadratic formula is

The first step in solving this problem is to substitute the numbers from the equation into the quadratic formula (keep in mind that the equation is in the form of ax2 + bx + c).

Next, simplify the problem to find the value of 222, which is 484.

Next, do the rest of the multiplication, as follows:

The square root of 484–484 is simply 0, so you can disregard it for the rest of the problem. You are left with:

 which simplifies to .

Because the ± does not give separate answers, there is only one answer to the problem:

 or −1.1.

2.   (4x2 − 7x + 3) − (10x2 + x − 11) = 0

4x2 − 7x + 3 − 10x2 − x + 11 = 0

-6x2 − 8x + 14 = 0

Multiply the entire equation by –1:

6x2 + 8x − 14 = 0

3.   For this problem, a = 4, b = 1, and c = −5. Substitute these numbers into the quadratic formula to get:

The square root of 81 is 9, so you now have:

Because of the ± sign, you have two possible answers. Find them by making two separate equations:

 and

Simplifying these two answers, you have your solutions: x = 1 and

4.   The first thing you must do is rearrange the equation to fit the format ax2 + bx + c = 0. After doing this, the equation will be 4x2 + 18x − 117. Therefore, the values for a, b, and c, respectively, are 4, 18, and –117.

5.   First, use FOIL to create a trinomial equation.

(2x + 4)2 = 0

(2x + 4)(2x + 4) = 0

4x2 + 8x + 8x + 16 = 0

4x2 + 16x + 16 = 0

Now use a = 4, b = 16, and c = 16 in the quadratic formula, as follows:

**Radical and Rational Expressions**

1.   In this problem, you are dealing with radicals. When it comes to radicals, an important rule to remember is that  Applying that rule to this question, you see that  The square root of 36 is 6.

2.   By rule,  Therefore,  Eliminate the radical in the denominator by multiplying the quantity by itself and repeating this multiplication on the numerator:

3.   This question shows what is called a “cube root.” The cube root of a number, x, is the number which raised to the third power gives x. This problem asks you to find the cube root of 27. Since 3 × 3 × 3 × is equal to 27, the cube root of 27 is 3.

4.   To answer this question, you must first multiply the two parts of the equation, as follows:

You can simplify this in order to find the square root:

Now that the problem is set up like this, the square root is clear:

5.   The rule used in this problem is.

Therefore,

**Inequalities and Absolute Value Equations**

1.   Since the inequality deals with an absolute value, |7x − 13| will always be a positive number. For the inequality to be true, 7x − 13 must be between the values of –22 AND 22. OR does not work here because the value must meet both the requirement of being larger than –22 as well as the requirement of being smaller than 22. If the absolute value is greater, use OR. If the absolute value is less than, use AND.

2.   To solve this problem, you must first drop the absolute value sign, and then create two separate inequalities, in the form of ax + b = c. The first inequality looks just like the original, while for the second one, you must switch the inequality sign and the sign of c, as follows:

It is impossible for a value to be greater than 7 AND less than –23. Therefore, use OR.

x > 7 OR x < −23.

3.   To solve this problem, you must drop the absolute value sign first, and then create two separate inequalities of the form ax + b = c. The first inequality looks just like the original, while for the second one, you must switch the inequality sign and the sign of c, as follows:

x must be less than 9 AND greater than –12. Unlike the previous problem, a number can meet both of these rules: x < 9 AND x > −12.

4.   To solve this problem, you must drop the absolute value sign first, and then create two separate inequalities, of the form ax + b = c. The first inequality looks just like the original, while for the second one, you must switch the inequality sign and the sign of c. A value cannot be both greater than 29 and less than –29, so OR must be used. Set up the two inequalities to find that 5x − 6 > 29 OR 5x − 6 < −29.

5.   To solve this problem, create two separate inequalities, as follows:

Because you multiplied both sides of each inequality by –4, you need to change the direction of the sign. Since x cannot be both less than –8 and greater than 32, OR is used: x < −8 OR x > 32.

**Sequences**

1.   In order to solve this problem, it is crucial to know the formula for arithmetic sequences. This formula is an = a1 + (n − 1)d, where an is the particular term you are trying to find, a1 is the first number in the sequence, and d is the common difference. This particular problem has already given you most of the information that you need. All that you have to do is substitute 3 for n, as you are looking for the 3rd term:

2.   This question asks you to write your own formula for the sequence. You will need the first term in the sequence, as well as the common difference. The first number is –8, and noticing that you jump from –8, to –2, and then to 4, it is clear that the common difference is 6. Your formula should look like this:

an = −8 + (n − 1)6

3.   In this problem, you are dealing with a geometric sequence. These sequences have a formula that looks like this: an = a1(r)n−1. Here, r is the constant ratio. Looking at the sequence, it goes from , to 1, to 4, and then to 16. This indicates that you must multiply by 4 each time; therefore 4 is the constant ratio. To find the 6th term in this sequence, you must set up the following formula:

4.   First of all, you need to find an answer that is similar to the formula used for an arithmetic sequence: an = a1 + (n − 1)d. Looking at the choices, you can eliminate the second and third because they are formulas for a geometric sequence. In the sequence you are given, the first term is 7, and the common difference is 6. Therefore, the correct answer is 7(8 − 1)(6).

5.   Here, you are asked to write your own formula once again. However, this time it is for a geometric sequence. The first term is 25, and you must also find the common ratio. To get from 25 to –5, you must divide by –5. This also works to get from –5 to 1, so the common ratio is –1/5. Your formula should look like this:

**Systems of Equations**

1.   When solving systems of equations, the best thing to do first is to isolate one of the variables. In this problem, you can do so by changing the sign on the bottom equation:

x − 2y = 14

−x + 4y = 8

Add the two equations together:

2y = 22

y = 11

Choose one of the original equations and substitute 11 for y. Solve for x.

x − 2(11) = 14

x − 22 = 14

x = 36

It is always a good idea to test your answers by substituting x and y values into both of the original equations.

2.   This problem is a little trickier than the first, as you cannot simply change the sign of one of the equations to isolate one of the variables. In this situation, you have to make the coefficients the same through multiplication. Since you know that 4 and 6 both go into 12, use the x term. Multiply the top equation by 3, and the bottom by 2:

12x − 6y = 18

−12x + 10y = 14

Add the two equations together:

4y = 32

y = 8

Finally, choose one of the original equations, substitute 8 for y, and solve for x.

4x − 2(8) = 6

4x − 16 = 6

4x = 22

, or

3.   The first step is rearranging the equations to align like terms:

3x − y = 18

4x + 6y = 24

Multiply the top equation by 6 and add the equations:

   Now choose one of the original equations, substitute 6 in for x, and solve for y:

3(6) − y = 18

18 − y = 18

−y = 0

y = 0

4.   First, distribute the 8 through the parentheses to get 8y + 8x = 12. You can then multiply the second equation by –2 to isolate one of the variables, and rearrange the equations to line up the like terms:

8x + 8y = 12

−8x + 6y = 44

Add the equations together:

14y = 56

y = 4

Now choose one of the original equations, substitute 4 for y, and solve for x.

8(x + 4) = 12

8x + 8(4) = 12

8x + 32 = 12

8x = −20

5.   First, line up the like terms in both equations:

4x − y = 63

x + 3y = 6

Multiply the top equation by 3 and add the equations.

12x − 3y = 189

x + 3y = 6

13x = 195

x = 15

Now substitute 15 for x in one of the equations.

x + 3y = 6

15 + 3y = 6

3y = −9

y = −3

**Logarithms**

1.   logx 27 = 3 means the log to the base x of 27 = 3. This means that x3 must equal 27, and therefore x must equal 3.

2.   By definition, loga b = c, if ac = b. In this question, you are asked to find the value of a. You are given the values of b and c, so your equation should look like this:

x4 = 625

You need to find a number that, when raised to the 4th power, equals 625. Test the answer choices: 44 = 256,74 = 2401, 54 = 625. Therefore, the correct answer is 5. You could immediately eliminate 7 after finding that 74 is already substantially larger than 625.

3.   To solve, turn the logarithm into an equation with an exponent:

3x = 729

Test some values for x:

32 = 9

33 = 27

34 = 81

35 = 243

36 = 729

Therefore, log 3 (729) = 6.

4.   By definition, logx 196 = 2 means the log to the base x of 196 = 2. This means that x2 must equal 196. To find the answer, you can simply take the square root of 196, which is 14.

5.   By definition, if log7 x = 3, then 73 = x. Therefore, x = 343.

**Roots of Polynomials**

1.   To solve this problem by factoring, you can start with a 2x on one side, and an x on the other:

(2x + / - \_\_\_)(x + / - \_\_\_)

These two missing numbers must add up to 9 (keep in mind that one of them is being multiplied by 2), and also must multiply to give –35. The only possible factors of 35 are 1, 5, 7, and 35. In looking at the problem, 5 and 7 seem like the most logical choices. You can try a few different combinations, but you should come up with:

(2x − 5)(x + 7)

To find the roots, set each quantity equal to 0:

2x − 5 = 0, x + 7 = 0

2x = 5, x = −7

2.   To solve this problem, begin with an x in both factors:

(x + / − \_\_)(x + / − \_\_)

The two missing numbers must have a sum of 2 and a product of –3. 3 is only divisible by 1 and 3, and the sum must be 2, so 3 is positive and 1 is negative.

(x − 1)(x + 3)

x − 1 = 0, x + 3 = 0

x = 1 and x = −3

3.   For this problem, you will have to work backward; you are already given the roots, and are being asked to find the equation to which they belong. Since the roots given are 6 and –2, you can write out x − 6 = 0 and x + 2 =0. Now, to find the original equation, you must multiply these two quantities:

(x − 6)(x + 2)

x2 − 6x + 2x − 12

x2 − 4x − 12

4.   To solve this problem, start with x in each of the factors:

(x + / − \_\_\_)(x + / − \_\_\_)

The sum of the missing numbers must be –8, and the product must be 16. Therefore, the numbers must both be –4.

(x − 4)(x − 4)

This can also be written (x − 4)2. Solve for x:

x − 4 = 0

x = 4

5.   Since the a value is –1, start with x and –x in the factors.

(x + / − \_\_\_)(−x + / − \_\_\_)

The sum must be 3 and the product must be 40, but remember that for the sum, one of the numbers is being multiplied by –1. In this case, 8 and 5 are the correct values:

(x + 5)(−x + 8)

x + 5 = 0 and −x + 8 = 0

x = −5 and x = 8

**Number Line Graphs**

1.   The answer is 8. Distance is always positive and can be shown as absolute value: |− 5 − 3 | = 8. You can also draw a number line, label –5 and 3, and see that the distance between those two points is 8.

2.   The midpoint is simply the point that is exactly halfway between the two points given. It can be thought of as an average. This value can be determined using the following formula:

Midpoint

Midpoint

Midpoint

Midpoint

3.   The answer is x ≥ −3 AND x < 6. AND is used because the bold line is connecting the two points. If there were a space, OR would be used. This eliminates the third choice. Open circles signify > or < and closed circles signify > or <. This eliminates the first and second choices.

4.   First, determine the values of x. Since both circles are open, > and < are used. Also, there is a space between the two points, so OR will be used. In the end, you have x < 2 OR x > 6. Now it is simply a matter of substituting the x values into the equations and determining which one is correct. The third choice, |2x – 10| > 6, is the correct answer.

5.   There is a space between the two points, so use OR. This eliminates the second and fourth answer choices. The third choice is incorrect because the graph does not show a bold line for values greater than –6. Also, the open circle means < or > needs to be used, as the values do not include –6. The first choice, x ≥ 2 OR x < –6, is correct.

**Equation of a Line and Slope of a Line**

1.   First, rearrange the equation into the slope-intercept form by isolating y. In this case, you divide by 2:

y = 2x + 6

In the slope-intercept formula, y = mx + b,b is the y-intercept. Because b = 6, the y-intercept is (0, 6).

2.   Rearrange the equation into the slope-intercept form by isolating y. In this case, you divide by 3:

You know that in the slope-intercept formula, y = mx + b,m is the slope.

Because  the correct answer is

3.   This equation represents a vertical line; the y-intercept is 0, so the line is parallel to the y-axis. A vertical line has an undefined slope. This is because slope is equivalent to “rise over run.” If the “run” is 0, the slope must be undefined because 0 cannot divide into anything.

4.   Remember that in the slope-intercept form y = mx + b,m is the slope and b is the y-intercept. In addition, parallel lines have the same slope; therefore, the slope of both lines (m) is 4. You are given that the y-intercept (the point at which the line crosses the y-axis) is 3. The equation of the line will be y = 4x + 3.

5.   First, rearrange the equation into slope-intercept form, by subtracting 3x and –y from both sides:

For two lines to be perpendicular, their slopes must be negative reciprocals. The negative reciprocal of  The problem also states that the perpendicular line has a y-intercept of 8. If you substitute  for m and 8 for b in the slope-intercept equation, you get

**Distance and Midpoint Formulas**

1.   The distance formula is: Distance

You can substitute the given values of x and y into the formula to solve for the distance, as follows:

2.   You can use the distance formula {Distance  solve this problem:

Take the square root of both sides.

15 = y − 8

23 = y

The following equation is also correct:

Square both sides.

Take the square root of both sides.

3.   Use the midpoint equation to solve this problem. First solve for the x-coordinate, which is half the distance between 12 and 10:

Do the same for ym, which is half the distance between 5 and –7:

Therefore, the midpoint is (11, –1).

4.   You only have to solve for the x-coordinate because you are given the y-coordinate:

5.   Use the distance formula: Distance =  to solve this problem:

**Properties and Relations of Plane Figures**

1.   The area of a triangle and the length of one of the legs of a right triangle are given. However, you need the length of both legs to use the Pythagorean Theorem to determine the hypotenuse. Since you have the area, start there. For a right triangle, Area  (base) × (height). You are given the base and area, so solve for the height:

Now you know the lengths of the two legs of the right triangle and can use the Pythagorean Theorem  to calculate the hypotenuse:

2.   The formula for the area of a circle is: area  The formula for the circumference of a circle is:  Since you are given the circumference, you can use that to find the radius, r, and then use the radius to find the area:

14π = 2πr

14 = 2r

r = 7

Now substitute r into the equation for area:

Area = π(72)

Area = 49π. The area of the circle is 49π in2.

3.   A parallelogram’s angles add up to 360°: 360° − 35° = 325°.

4.   The equation for the area of a trapezoid is: area  (base1 + base2) (height). Substitute the given variables into the equation and solve for the missing base:

5.   A square is a special kind of rectangle. All of its sides are equal in length. Since the area of a rectangle is area = l × w, the area of a square would be area = s2 (side squared) because length and width are equal. For this problem, the given side is 6 mm. If the figure were a square, the area would be 36 mm2. However, the area is said to be 42 mm2. Therefore the shape is a rectangle and not a square.

**Angles, Parallel Lines, and Perpendicular Lines**

1.   Supplementary angles add together to total 180°. Therefore, the supplementary angle to a 40° angle is a 140° angle.

2.   Supplementary angles add together to total 180°. Therefore the supplementary angle to a 25° angle is a 155° angle.

3.   The 90° angle marked indicates that the other three angles formed by the intersection of lines p and o each measure 90° also. As line n is parallel to line m, the same four 90° angles are formed at the intersection of lines p and m. Similarly, the angles on line o each measure 90°, too, because lines p and o are parallel. Thus, angle θ = 90°.

4.   The transversal crosses two parallel lines, so the angles made at the intersections will be identical. 43° corresponds to the supplementary angle of a on line y. Since 43° and a are supplementary angles, they must add up to 180°. Therefore, the answer is 180° − 43° = 137°.

5.   Since line v is perpendicular to line t, it forms four right angles. The line segment that is unnamed in the diagram dissects one of the right angles. Angle a is one side and 35° is the measurement given for the other side. These two angles add up to 90° : 90° − 35° = 55°. Therefore, the angle measures 55°.

**Perimeter, Area, and Volume**

1.   The question asks you to determine the number of bags of fertilizer that will cover your rectangular backyard. According to information in the problem, 6 pounds of fertilizer can cover 700 square feet. Begin by calculating the area of the rectangular backyard. The area of a rectangle is determined by multiplying the length (70 feet) by the width (40 feet):

70 × 40 = 2,800

The area of the rectangular backyard is 2,800 square feet. The problem states that 6 pounds of fertilizer can cover 700 square feet. Calculate the number of times that 700 will go into 2,800:

2,800 ÷ 700 = 4

You will need 4 times 6 pounds of fertilizer to treat 2,800 square feet:

4 × 6 = 24

Since you will need a total of 24 pounds of fertilizer to treat the backyard, and each bag of fertilizer weighs 8 pounds, divide 24 by 8 to find the number of bags of fertilizer you will need:

24 ÷ 8 = 3

You will need 3 bags of fertilizer to treat a backyard that measures 2,800 square feet.

2.   If the pool has a diameter of 26 feet, and the fence needs to be 4 feet away from the edge of the pool, the diameter of the area enclosed by the fence would be 26 + 4 + 4 = 34 feet. Draw a picture to help visualize the problem:

The area of a circle is πr2. The radius is half of the diameter, so r = 17. Substitute 17 for r and 3.14 for π and solve:

Area = (3.14)(17)2

Area = 907.46 ft2

3.   A beach ball is a sphere, and the formula for the volume of a sphere is: . The diameter is given as 0.6 m, so the radius is half of that, 0.3 m. Substitute that value into the formula and compute the volume:

4.   The formula for the volume of a cylinder is πr2h. The question is asking for diameter, so first solve for r, then double it.

Since the radius is 5 cm, the diameter is 10 cm.

5.   The equation for the volume of a cube is: s3. Since we are given an edge, or side (s) of 5, you simply substitute 5 for s. The answer is 125 in3.

**Trigonometry**

1.   Using the mnemonic SOHCAHTOA helps you remember that sine is the ratio of “opposite to hypotenuse.” The side opposite of a has a length of 12. The hypotenuse has a length of 13. So, sin

2.   Using the mnemonic SOHCAHTOA helps you remember that tangent is the ratio of “opposite to adjacent” and cosine is “adjacent over hypotenuse.” Since you are given cosine, you know the lengths of two sides of the right triangle. The adjacent leg is 4 and the hypotenuse is 5. Using the Pythagorean Theorem (a2 + b2 = c2), you can calculate the length of the opposite leg, and then calculate tan a:

a2 + 42 = 52

a2 + 16 = 25

a2 = 9

a = 3

Now you have the adjacent (4) and opposite (3) legs, so tan

3.   By definition, to convert degrees to radians multiply by :

4.   To convert radians to degrees

5.   By definition, secant is the reciprocal of cosine, which is calculated by dividing the length of the adjacent side by the length of the hypotenuse (adj/hyp). Therefore, cos a = 5 / 13, and the length of the side adjacent to the angle is 5, while the length of the hypotenuse is 13. By definition, sine is equivalent to opposite/hypotenuse, so you must use the Pythagorean Theorem (a2+ b2 = c2) to find the length of the side opposite angle a:

a2 + 52 = 132

a2 + 25 = 169

a2 = 144

a = 12

Because a = 12, the sin of angle

**Translating Word Problems**

1.   You are given that Tom started out with 6 books. After he gave 2 books to his sister he was left with 6 – 2 books. He then purchased 3 more books, so he now has 6 – 2 + 3 books.

2.   To solve this problem, start with William and work backward. William walked 2 miles, and Rebecca walked 4 times as far as William. Therefore, Rebecca walked 4(2) miles. Juan walked 3 more miles than Rebecca, so Juan walked 4(2) + 3 miles.

3.   The first step is to calculate the total cost of the CDs: 2(15) = 30. You are given that, in addition to the 2 CDs, Tina also purchases d of the DVDs, each of which costs $18. Therefore, her cost for the DVDs was 18d. Now simply add the terms together to get 30 + 18d.

4.   You are given that Mark, m, is older than Frank, f. Therefore, f < m. You are also given that Mark, m, is younger than David, d. Therefore, m < d. Mark’s age is between Frank and David’s ages, so f < m < d.

5.   You are given that Jim is j years old today; therefore, 2 years ago, Jim would have been j – 2 years old. At that time, Kathy was twice as old as Jim, or 2( j – 2).